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Multiple membranes from gauged supergravity

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Abstract: Starting from gauged $\mathcal{N} = 8$ supergravity in three dimensions we construct actions for multiple membranes by taking the limit to global supersymmetry for different choices of the embedding tensor. This provides a general framework that reproduces many recent results on multiple membrane actions as well as generalisations thereof. As examples we discuss conformal (non-conformal) gaugings leading to multiple M2-branes (D2-branes) and massive deformations of these systems.

Keywords: Supersymmetric gauge theory, Field Theories in Lower Dimensions, D-branes, Chern-Simons Theories.

Contents

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1. Introduction

Recently, there has been a lot of activity in constructing actions for multiple M2-branes. This development was spurred by a series of papers by Bagger and Lambert [1 – 3] and Gustavsson [4, 5] (following earlier work of [6, 7]) who made a proposal for a three-dimensional action describing multiple M2-branes. This action is an $\mathcal{N} = 8$ superconformal Chern-Simons gauge theory.

It turns out that the original proposal of [1 – 5] is rather restrictive. The presence of a so-called fundamental identity leads to a basically unique solution with SO(4) gauge group (or direct sums thereof) [8, 9] that describes a system of two M2-branes on an orbifold [10, 11]. To describe more general M2-brane systems an extension of the original proposal is needed and several extensions have been considered. A possibility is to consider supersymmetric gauge theories without a Lagrangian [12]. Also massive extensions breaking the conformal invariance have been constructed [13 – 15]. More recently, new extensions to arbitrary gauge groups of the Bagger-Lambert theory have been proposed that make use of an invariant metric that is not positive definite [16 – 18]. This has the potentially troublesome feature that it introduces ghosts, an issue which has been addressed in [19 – 21]. In addition, Chern-Simons theories with less supersymmetries in the context of M2-branes have been considered [22, 23]. For other related work on multiple M2-branes, see [24].

The recent interest in multiple membranes deals with the properties of globally $\mathcal{N} = 8$ supersymmetric gauge theories in three dimensions. Independent of this, a lot is known about the construction of *locally* $\mathcal{N} = 8$ supersymmetric theories in three dimensions. There are a few parallel developments in constructing theories with global versus local supersymmetry. For instance, one issue with the construction of an $\mathcal{N} = 8$ supersymmetric gauge theory in three dimensions is the origin of the gauge fields. To describe M2-brane actions one needs to work with the maximum number $8N$ of scalar kinetic terms and there is no room for a vector field kinetic term. The way out was given in [6]. The vector fields needed for the gauging only occur inside the covariant derivatives and via a Chern-Simons term but do not have a kinetic term. Their field equations lead to a duality relation between the vectors and the scalars such that no new degrees of freedom are introduced. Precisely the same issue was encountered in the construction of gauged supergravity in three dimensions [25 – 27]. For instance, in $\mathcal{N} = 8$ supergravity all bosonic degrees of freedom are described by scalars parametrizing the coset $\mathrm{SO}(8, N)/(\mathrm{SO}(8) \times \mathrm{SO}(N))$, and there are no vector fields left to perform the gauging. The resolution proposed in [25, 26] is the same as in the globally supersymmetric case: the vector fields only occur via covariant derivatives and a Chern-Simons term.

A noteworthy feature of gauged supergravities in three dimensions is that it suffices to restrict oneselves to theories in which the Yang-Mills gauge fields only occur via a Chern-Simons term without a separate kinetic term. This is due to the existence of a non-Abelian duality which states that any Yang-Mills theory in three dimensions can be re-interpreted as the sum of kinetic terms for scalar fields and a $B \wedge F(A)$ Chern-Simons gauge theory (containing two distinct vector fields A and B) based on a non-semi-simple Lie algebra [28, 29]. It is via such Chern-Simons terms that we recover, after applying the non-Abelian duality, results for multiple D2-brane actions as well. We will also encounter Chern-Simons gauge theories of the type $A \wedge F(A)$, which are topologically massive gauge theories.

The construction of [25, 26] classifies the most general gaugings in supergravity, which are encoded in the 'embedding tensor'. The role of this tensor is to specify which subgroup of the global symmetry group is gauged and which vectors are needed to perform this gauging. Originally this technique was developed to construct maximal supergravity in three dimensions and was later applied to the $\mathcal{N} = 8$ case [27, 29] and in higher dimensions as well [30 – 37]. The same technique can be applied to supersymmetric gauge theories. This has been done to construct the gaugings of $\mathcal{N} = 2$ supersymmetry in four dimensions [38] and, more recently, to reconstruct [39] the supersymmetric gauge theory of [1 – 5]. In the latter case the embedding tensor is a 4-index anti-symmetric tensor of $\mathrm{SO}(N)$ that coincides with the 'structure constants' of the three-algebra occurring in the construction of [1 – 5].

In contrast to the supersymmetric gauge theory with the unique $\mathrm{SO}(4)$ gauge group of [1 – 5], in supergravity a wide variety of gaugings is possible. In particular, one can embed the gauge group into the *non-compact group* SO$(8,N)$ whereas only subgroups of the *compact* SO(N) group are gauged in [1–5]. In this work we want to investigate the relation between the two types of theories and their gaugings. In particular we want to address the following question: starting from $\mathcal{N}=8$ gauged supergravity in three dimensions, can we take the limit of global supersymmetry and if so, does this lead to known and/or new supersymmetric gauge theories describing multiple branes?

In order to answer this question we have organised this paper as follows. In section 2 we will first write down the $\mathcal{N}=8$ supergravity theory and next consider the limit to global supersymmetry. Furthermore, we will present the general result for the globally supersymmetric theory. In section 3 we focus in on the separate deformations and discuss their interpretation in terms of multiple branes. Our conclusions are presented in section 4. Finally, appendix A contains our conventions and useful formulae for the SO$(8,N)$ structure of supergravity.

2. Gauged $\mathcal{N}=8$ supergravity and its global limit

2.1 The Lagrangian and the embedding tensor

We start by reviewing $\mathcal{N}=8$ gauged supergravity in $D=3$ [27, 29]. For an overview of our conventions see appendix A.

The $\mathcal{N}=8$ supergravity multiplet consists of the metric $g_{\mu\nu}$ and 8 gravitini ψ_μ^A. All these fields are topological and do not describe physical degrees of freedom. Therefore, all local degrees of freedom reside in scalars and Majorana spinor fields. In the case of N matter multiplets, there are $8N$ scalars X^{aI}, $a=1,\ldots,N$, parameterizing the coset space SO$(8,N)/($SO$(8)\times$ SO$(N))$, and $8N$ spinors denoted by $\chi^{\dot{A}a}$. The coset dynamics of the scalar fields is expressed in terms of the group-valued matrix $L(x)\in$ SO$(8,N)$. It can be parameterized in terms of scalars in the following way

$$L(x) \;=\; \exp\left(X^{Ia}(x)t^{Ia}\right) \,, \tag{2.1}$$

where $\{t^{IJ}, t^{ab}\}$ and $\{t^{Ia}\}$ denote the compact and non-compact generators of $\mathfrak{so}(8,N)$, c.f. the appendix. To be more precise, we have gauge-fixed the local SO$(8)\times$ SO(N) symmetry by setting the compact part of L to zero.

In order to gauge a certain subgroup G_0 of the (rigid) duality group SO$(8,N)$ one introduces gauge-covariant derivatives in the definition of the Maurer-Cartan forms as follows:

$$L^{-1}\left(\partial_\mu + \Theta_{\alpha\beta}A_\mu{}^\alpha t^\beta\right)L \;=:\; \frac{1}{2}\mathcal{Q}_\mu{}^{IJ}t^{IJ} + \frac{1}{2}\mathcal{Q}_\mu{}^{ab}t^{ab} + \mathcal{P}_\mu{}^{Ia}t^{Ia} \,. \tag{2.2}$$

Here t^α denote the generators of SO$(8,N)$, $\alpha,\beta,\ldots = 1,\ldots,\frac{1}{2}(N{+}8)(N{+}7)$, with structure constants $f^{\alpha\beta}{}_\gamma$. Furthermore, we have introduced gauge fields $A_\mu{}^\alpha$ in the adjoint representation of $\mathfrak{so}(8,N)$ and the symmetric embedding tensor $\Theta_{\alpha\beta}$ [27]. The latter encodes the embedding of the gauge group G_0 into the global symmetry group SO$(8,N)$. To be more precise, the generators of G_0 are given by

$$X_\alpha = \Theta_{\alpha\beta}t^\beta \,, \tag{2.3}$$

and so the embedding tensor singles out those generators that span the gauge group. In particular, the dimension of the gauge group is determined by the rank of $\Theta_{\alpha\beta}$.

The gauged supergravity Lagrangian is completely determined by the embedding tensor and given by [27]

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}\kappa^{-2}e\,R + \varepsilon^{\mu\nu\rho}\bar{\psi}^A_\mu D_\nu\psi^A_\rho + \frac{1}{2}\kappa^{-2}e\,\mathcal{P}_\mu{}^{Ia}\mathcal{P}^{\mu\,Ia} - ie\,\bar{\chi}^{\dot{A}a}\gamma^\mu D_\mu\chi^{\dot{A}a} \\ &-\frac{1}{2}\Theta_{\alpha\beta}\varepsilon^{\mu\nu\rho}A_\mu{}^\alpha\left(\partial_\nu A_\rho{}^\beta + \frac{1}{3}\Theta_{\gamma\delta}f^{\beta\delta}{}_\epsilon A_\nu{}^\gamma A_\rho{}^\epsilon\right) - e\,\mathcal{P}_\mu{}^{Ia}\bar{\chi}^{\dot{A}a}\Gamma^I_{A\dot{A}}\gamma^\nu\gamma^\mu\psi^A_\nu \\ &+\kappa^{-2}e\,A_1^{AB}\bar{\psi}^A_\mu\gamma^{\mu\nu}\psi^B_\nu + 2i\kappa^{-2}e\,A_2^{A,\dot{A}a}\bar{\chi}^{\dot{A}a}\gamma^\mu\psi^A_\mu + \kappa^{-2}e\,A_3^{\dot{A}a,\dot{B}b}\bar{\chi}^{\dot{A}a}\chi^{\dot{B}b} - \kappa^{-6}e\,V\,,\end{aligned} \tag{2.4}$$

where κ is the square root of Newton's constant with mass dimension[1] $-\frac{1}{2}$ and γ^μ and $\Gamma^I_{A\dot{A}}$ are gamma matrices of SO(1, 2) and SO(8), respectively. Furthermore, the covariant derivatives D_μ on the spinors are given by

$$\begin{aligned}D_\mu\psi^A_\nu &= \nabla_\mu\psi^A_\nu + \frac{1}{4}\mathcal{Q}^{IJ}_\mu\Gamma^{IJ}_{AB}\psi^B_\nu\,, \\ D_\mu\chi^{\dot{A}a} &= \nabla_\mu\chi^{\dot{A}a} + \frac{1}{4}\mathcal{Q}^{IJ}_\mu\Gamma^{IJ}_{\dot{A}\dot{B}}\chi^{\dot{B}a} + \mathcal{Q}^{ab}_\mu\chi^{\dot{A}b}\,,\end{aligned} \tag{2.5}$$

and contain the (composite) SO(8)×SO(N) connections defined in (2.2). Finally, the scalar-dependent Yukawa couplings given by $A_{1,2,3}$ and the scalar potential V are completely determined by the embedding tensor via the so-called T-tensor

$$T_{\underline{\alpha},\underline{\beta}} = \Theta_{\alpha\beta}\mathcal{V}^\alpha{}_{\underline{\alpha}}\mathcal{V}^\beta{}_{\underline{\beta}}\,, \tag{2.6}$$

where $\underline{\alpha},\underline{\beta},\dots$ are flat indices corresponding to the local SO(8) × SO(N) action. Here $\mathcal{V}$ is the adjoint SO(8, N) matrix, defined through

$$L^{-1}t^\alpha L = \mathcal{V}^\alpha{}_{\underline{\alpha}}t^{\underline{\alpha}} = \frac{1}{2}\mathcal{V}^\alpha{}_{IJ}t^{IJ} + \frac{1}{2}\mathcal{V}^\alpha{}_{ab}t^{ab} + \mathcal{V}^\alpha{}_{Ia}t^{Ia}\,. \tag{2.7}$$

In terms of the T-tensor the Yukawa couplings read

$$\begin{aligned}A_1^{AB} &= -\delta^{AB}\theta - \frac{1}{48}\Gamma^{IJKL}_{AB}T_{IJ,KL}\,, \qquad A_2^{A,\dot{A}a} = -\frac{1}{12}\Gamma^{IJK}_{A\dot{A}}T_{IJ,Ka}\,, \\ A_3^{\dot{A}a,\dot{B}b} &= 2\delta^{\dot{A}\dot{B}}\delta^{ab}\theta + \frac{1}{48}\delta^{ab}\Gamma^{IJKL}_{\dot{A}\dot{B}}T_{IJ,KL} + \frac{1}{2}\Gamma^{IJ}_{\dot{A}\dot{B}}T_{IJ,ab}\,,\end{aligned} \tag{2.8}$$

where $\theta = \frac{2}{(N+8)(N+7)}\eta^{\alpha\beta}\Theta_{\alpha\beta}$ denotes the trace of the embedding tensor with respect to the Cartan-Killing form $\eta^{\alpha\beta}$ of SO(8, N). The scalar potential V is given by

$$V = -\frac{1}{2}\left(A_1^{AB}A_1^{AB} - \frac{1}{2}A_2^{A,\dot{A}a}A_2^{A,\dot{A}a}\right)\,. \tag{2.9}$$

The local supersymmetry transformations leaving (2.4) invariant are given by

$$\begin{aligned}\delta_\epsilon e_\mu{}^r &= i\kappa\,\epsilon^A\gamma^r\psi^A_\mu\,, \qquad \delta_\epsilon\psi^A_\mu = \kappa^{-1}D_\mu\epsilon^A + i\kappa^{-3}A_1^{AB}\gamma_\mu\epsilon^B\,, \\ \delta_\epsilon A_\mu{}^\alpha &= -\frac{1}{2}\kappa^{-1}\mathcal{V}^\alpha{}_{IJ}\bar{\epsilon}^A\Gamma^{IJ}_{AB}\psi^B_\mu + i\kappa^{-1}\mathcal{V}^\alpha{}_{Ia}\bar{\epsilon}^A\Gamma^I_{A\dot{A}}\gamma_\mu\chi^{\dot{A}a}\,, \\ \delta_\epsilon\chi^{\dot{A}a} &= \frac{1}{2}i\kappa^{-1}\Gamma^I_{A\dot{A}}\gamma^\mu\epsilon^A\mathcal{P}_\mu{}^{Ia} + \kappa^{-3}A_2^{A,\dot{A}a}\epsilon^A\,, \qquad L^{-1}\delta_\epsilon L = \kappa\,\bar{\epsilon}^A\Gamma^I_{A\dot{A}}\chi^{\dot{A}a}t^{Ia}\,,\end{aligned} \tag{2.10}$$

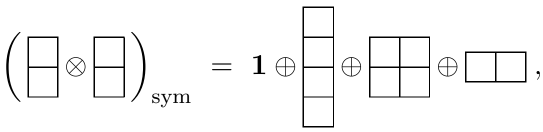
[1]The mass dimensions of $\{g_{\mu\nu}, \psi^A_\mu, A_\mu{}^\alpha, \chi^{\dot{A}a}, X^{Ia}\}$ are given by $\{0, 1, 1, 1, 0\}$.

where we assign mass dimension $-\frac{1}{2}$ to the supersymmetry parameter ϵ.

However, consistency of the gauged supergravity theory requires linear and quadratic constraints on the embedding tensor. First of all, gauge invariance of (2.4) requires invariance of the embedding tensor $\Theta_{\alpha\beta}$ under the adjoint action of the gauge group generators X_α. This implies the quadratic constraint

$$\mathcal{Q}_{\alpha,\beta\gamma} \;\equiv\; \Theta_{\alpha\delta}\Theta_{\epsilon(\beta}f^{\delta\epsilon}{}_{\gamma)} \;=\; 0\,, \tag{2.11}$$

which is also sufficient for closure of the gauge algebra. Beyond this quadratic constraint, invariance of (2.4) under supersymmetry requires a linear constraint, which takes a "duality covariant" form. The embedding tensor reads $\Theta_{\alpha\beta} = \Theta_{[\mathcal{I}\mathcal{J}],[\mathcal{K}\mathcal{L}]}$, $\mathcal{I}, \mathcal{J}, \ldots = 1, \ldots, 8+N$, where we have introduced adjoint indices for $\mathrm{SO}(8,N)$. Due to its symmetry, a priori it takes values in the symmetric tensor product



(2.12)

in which the Young tableaux refer to tensors of the full duality group $\mathrm{SO}(8,N)$. However, supersymmetry restricts the irreducible representations in (2.12) to a subclass. Specifically, in the given case it eliminates the irreducible representation corresponding to the window tableau [29]. In other words, the linear constraint can be written as

$$\Theta_{\mathcal{I}\mathcal{J},\mathcal{K}\mathcal{L}} \;=\; \theta\delta_{\mathcal{I}[\mathcal{K}}\delta_{\mathcal{L}]\mathcal{J}} + 2f_{\mathcal{I}\mathcal{J}\mathcal{K}\mathcal{L}} + h_{[\mathcal{K}[\mathcal{I}}\delta_{\mathcal{J}]\mathcal{L}]}\,, \tag{2.13}$$

where $f_{\mathcal{I}\mathcal{J}\mathcal{K}\mathcal{L}}$ is totally antisymmetric and $h_{\mathcal{I}\mathcal{J}}$ symmetric-traceless. For any choice of the embedding tensor satisfying the quadratic and linear constraints (2.11) and (2.13) one obtains a consistent gauged supergravity, which is invariant under the supersymmetry transformations (2.10).[2]

2.2 The limit to global supersymmetry

We will now discuss the limit to global supersymmetry, i.e. we decouple gravity by sending Newton's constant, or its square root κ, to zero. We will find that this limit can only be taken provided a number of additional constraints is imposed on the embedding tensor.

To take the flat space limit, we have to expand the metric around Minkowski spacetime according to

$$g_{\mu\nu} \;=\; \eta_{\mu\nu} + \kappa h_{\mu\nu}\,. \tag{2.14}$$

In the limit $\kappa \to 0$, the spin-2 multiplet $\{h_{\mu\nu}\,, \psi^A_\mu\}$ decouples and can therefore be set to zero. This restricts the parameters ξ_μ of general coordinate transformations and the parameters ϵ^A of supersymmetry to those satisfying the equations $\partial_\mu\,\xi_\nu + \partial_\nu\,\xi_\mu = \partial_\mu\epsilon^A = 0$. Thus, we obtain a globally supersymmetric theory, in which ϵ^A is constant. Moreover, in

[2] We should note that the expressions (2.8) for the Yukawa couplings are valid provided the embedding tensor satisfies the stronger constraint $h_{\mathcal{I}\mathcal{J}} = 0$. We verified that in the presence of $h_{\mathcal{I}\mathcal{J}}$ one can still take the global limit to be discussed below. For the general formulae see [29].

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order to obtain a non-singular limit, in which non-trivial gaugings survive, it turns out to be necessary to rescale the fields and various components of the embedding tensor with powers of κ. Specifically, we redefine the scalar fields according to

$$X^{Ia} \to \kappa\, X^{Ia}\,, \tag{2.15}$$

such that their mass dimension is $\frac{1}{2}$, and we redefine the gauge vectors depending on their $\mathrm{SO}(8)\times\mathrm{SO}(N)$ indices according to

$$A_\mu{}^{ab} \to A_\mu{}^{ab}\,, \qquad A_\mu{}^{aI} \to \kappa^{-1} A_\mu{}^{aI}\,, \qquad A_\mu{}^{IJ} \to \kappa^{-1} A_\mu{}^{IJ}\,. \tag{2.16}$$

Moreover, we require the following scaling weights for the components of the embedding tensor,

$$\Theta_{ab,cd} \,:\, 0\,, \qquad \Theta_{ab,cI}\,, \Theta_{ab,IJ} \,:\, 1\,, \qquad \Theta_{aI,cJ}\,, \Theta_{aI,JK}\,, \Theta_{IJ,KL} \,:\, 2 \tag{2.17}$$

where we indicated the powers of κ.

Let us now explain the limit and the origin of the different scaling weights in more detail. First of all, inspection of the scalar-kinetic terms shows by use of the expansion of the Maurer-Cartan forms (A.6) that the terms of higher order in X will drop out. In other words, the scalar manifold reduces to a flat space. This can be interpreted as an Inönü-Wigner contraction of the original coset space, for which one rescales the non-compact generators by $\bar{t}^{Ia} = \kappa t^{Ia}$ and sends $\kappa \to 0$. This leaves the algebra, see eq. (A.2), intact, except the brackets in the last line, which become Abelian. Put differently, the Lie algebra reduces to a semi-direct product between $\mathrm{SO}(8)\times\mathrm{SO}(N)$ and $8N$ translations. The coset space reduces accordingly to

$$\frac{(\mathrm{SO}(8)\times\mathrm{SO}(N)) \ltimes \mathbb{R}^{8N}}{\mathrm{SO}(8)\times\mathrm{SO}(N)} \cong \mathbb{R}^{8N}\,. \tag{2.18}$$

Note that the isometry group $I\,\mathrm{SO}(8N)$ of $\mathbb{R}^{8N}$ is much larger than the expected symmetry group $(\mathrm{SO}(8)\times\mathrm{SO}(N)) \ltimes \mathbb{R}^{8N}$. However, this symmetry enhancement only holds for the scalar kinetic terms, and does not extend to the full theory.

The scaling weights of the gauge fields are uniquely determined by requiring that the supersymmetry transformations of the vectors in (2.10) are both non-singular and non-zero in the limit $\kappa \to 0$. For instance, one finds

$$\delta_\epsilon A_\mu{}^{IJ} = -\bar{\epsilon}^A \Gamma^{IJ}_{AB}\, \psi^B_\mu\,. \tag{2.19}$$

One may verify that the global supersymmetry algebra is realized on these vector fields provided that the following constraints are satisfied:[3]

$$\varepsilon^{\mu\nu\rho}\partial_\nu A_\rho{}^{IJ} = 0\,. \tag{2.20}$$

[3]Similarly, it has recently been found that one can realize the supersymmetry algebra of $D=5, \mathcal{N}=2$ supergravity on $(D-2)$-forms with vanishing field strengths [40].

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We note that the supersymmetry variation of these gauge vectors is proportional to the gravitino. Therefore they belong to the topological spin-2 multiplet $\{g_{\mu\nu}\,,\psi_\mu^A\,,A_\mu{}^{IJ}\}$ and we will henceforth set them to zero. In contrast, we will see below that the other two representations of gauge vectors in (2.16) are related to the matter spinors χ under supersymmetry. Therefore they belong to the matter multiplets and will make their appearance in the globally supersymmetric theory. We also note that the scaling weights (2.17) lead to a non-singular limit for the leading Chern-Simons terms, which otherwise would require certain components of the embedding tensor to vanish.

Let us now turn to the constraints of the embedding tensor describing globally supersymmetric theories. The linear constraints (2.13) ensure that the gauged supergravity action is a consistent starting point. However, in order to have a well-defined limit further constraints are required, which we derive by inspecting the Yukawa couplings A_i with $i=1,2,3$. We first consider the scalar potential. To avoid any divergent terms, both A_1 and A_2 have to scale with weight 3. Turning to the supersymmetry variation of the gravitino, the right hand side only vanishes if A_1 actually scales with weight 4. This has the effect that A_1 completely drops out of the theory in the global limit, as expected. Finally, the scaling weight of A_3 has to be 2, as can be deduced from the relevant term in the Lagrangian. We thus end up with the following scaling weights for A_1, A_2 and A_3:

$$A_1\ :\ 4\,,\qquad\qquad A_2\ :\ 3\,,\qquad\qquad A_3\ :\ 2\,. \tag{2.21}$$

From the expressions (2.8), (A.8) for A_1, A_2 and A_3 we deduce that the above scaling requirements lead to the following linear constraints on the embedding tensor:

$$\theta=0\,,\qquad \Theta_{ab,IJ}=0\,,\qquad \Theta^-_{IJ,KL}=0\,,\qquad \Theta_{a[I,JK]}=0\,, \tag{2.22}$$

where $\Theta^-_{IJ,KL}$ denotes the anti-self-dual part of $\Theta_{IJ,KL}$.

Apart from the constraints (2.22) resulting from the requirement of a non-singular limit, there is a second source of linear constraints. This is related to the fact that the original linear constraint (2.13) of supergravity cannot simply be taken over to the globally supersymmetric case due to the following reason. The symmetric-traceless solution $h_{\mathcal{I}\mathcal{J}}$, for instance, in general gives rise to components of the embedding tensor that scale differently. For instance, if $h_{\mathcal{I}\mathcal{J}}$ takes non-zero values only in the SO(N) direction, parameterized by a symmetric-traceless SO(N) tensor h_{ab}, one obtains from (2.13) the components[4]

$$\Theta_{ab,cd}\ =\ h_{[a[c}\delta_{d]b]}\ ,\qquad \Theta_{aI,bJ}\ =\ \frac{1}{4}h_{ab}\delta_{IJ}\ . \tag{2.23}$$

Since according to (2.17) we keep the first component unchanged, while rescaling the second embedding tensor by κ^2, the resulting couplings live in different sectors characterized by embedding tensors of different mass dimension. In general the supersymmetry will therefore be violated. Thus, in order to maintain supersymmetry, we have to impose additional linear constraints, eliminating all solutions of (2.13) which give rise to relations between different components of Θ with different scaling weights, like in (2.23). This sets the singlet θ to

[4]We thank Hermann Nicolai for discussions on this point.

zero, which follows already from (2.22), as well as the components f_{IJab} of the 4-index anti-symmetric tensor and the components h_{ab} and h_{Ia} of the symmetric-traceless tensor.

Summarizing, we find from the above considerations that the only components of the embedding tensor that survive the limit of global supersymmetry are given by

$$f_{abcd}\,, \qquad f_{abcI}\,, \qquad f^{+}_{IJKL}\,, \qquad h_{IJ}\,, \tag{2.24}$$

where f^{+}_{IJKL} indicates the self-dual part of f_{IJKL}.

Finally, we consider the quadratic constraints. One way to derive these constraints after the rescalings is to consider the gauge variation of the action before imposing any constraints. For instance, the Chern-Simons term varies according to [39, 41]

$$\delta\mathcal{L}_{\rm CS} \;\sim\; \varepsilon^{\mu\nu\rho}\mathcal{Q}_{\alpha,\beta\gamma}A_\mu{}^\alpha A_\nu{}^\beta D_\rho\Lambda^\gamma\,. \tag{2.25}$$

Suppose the gauge vectors and their symmetry parameters in $AAD\Lambda$ will scale with κ^{-r} for some r, as follows from (2.16). Then only those parts of $\mathcal{Q}_{\alpha,\beta\gamma}$ will not disappear in the limit $\kappa\to 0$, whose dependence on κ is κ^s with $s\leq r$. For instance, since the SO(N) gauge vectors do not scale with κ, only those terms in $\mathcal{Q}_{ab,cd,ef}$ should be imposed as a constraint that scale with κ^0. This in turn implies that in the latter component only the pure SO(N) structure constants enter, while in the full quadratic constraints of supergravity also the non-compact $f^{aI,bJ}{}_{cd}$ appear. Similarly, one derives for all other components that the non-trivial parts of the quadratic constraint tensor involve only the structure constants corresponding to SO$(N)\ltimes\mathbb{R}^{8N}$. In other words, denoting these structure constants, i.e. $f^{ab,cd}{}_{ef}$ and $f^{ab,cI}{}_{dJ}$, collectively by $\bar{f}^{\alpha\beta}{}_\gamma$, the quadratic constraints imposed by global supersymmetry take formally the same form as in (2.11), but with f replaced by $\bar{f}$,

$$\mathcal{Q}_{\alpha,\beta\gamma} \;\equiv\; \Theta_{\alpha\delta}\Theta_{\epsilon(\beta}\bar{f}^{\delta\epsilon}{}_{\gamma)} \;=\; 0\,. \tag{2.26}$$

Moreover, since we set $A_\mu{}^{IJ}$ and its gauge parameter to zero, all components of (2.26) whose external indices take values in the $[IJ]$ direction, need not to be imposed as constraints. Explicitly one then finds the following non-trivial components:

$$\begin{aligned}
\mathcal{Q}_{ab,cd,ef} &= \frac{1}{2}\big(\Theta_{ab,eg}\Theta^g{}_{f,cd}-\Theta_{ab,fg}\Theta^g{}_{e,cd}+\Theta_{ab,cg}\Theta^g{}_{d,ef}-\Theta_{ab,dg}\Theta^g{}_{c,ef}\big)\,, &(2.27)\\
\mathcal{Q}_{aI,bJ,cd} &= \frac{1}{2}\left(\Theta_{aI,c}{}^g\Theta_{gd,bJ}-\Theta_{aI,d}{}^g\Theta_{gc,bJ}-\Theta_{aI,gb}\Theta^g{}_{J,cd}\right)\,, &(2.28)\\
\mathcal{Q}_{ab,cd,eI} &= \frac{1}{2}\left(\Theta_{ab,eg}\Theta^g{}_{I,cd}+\Theta_{ab,gI}\Theta^g{}_{e,cd}+\Theta_{ab,cg}\Theta^g{}_{d,eI}-\Theta_{ab,dg}\Theta^g{}_{c,eI}\right)\,, &(2.29)\\
\mathcal{Q}_{aI,bc,de} &= \frac{1}{2}\Big(\Theta_{aI,d}{}^h\Theta_{he,bc}-\Theta_{aI,e}{}^h\Theta_{hd,bc}+\Theta_{aI,b}{}^h\Theta_{hc,de}-\Theta_{aI,c}{}^h\Theta_{hb,de}\Big)\,, &(2.30)\\
\mathcal{Q}_{ab,cI,dJ} &= \Theta_{ab,c}{}^e\Theta_{eI,dJ}+\Theta_{ab,d}{}^e\Theta_{eJ,cI}\,, &(2.31)\\
\mathcal{Q}_{aI,bJ,cK} &= \Theta_{aI,b}{}^d\Theta_{dJ,cK}+\Theta_{aI,c}{}^d\Theta_{dK,bJ}\,, &(2.32)
\end{aligned}$$

where all indices are raised and lowered with the ordinary Kronecker symbol.

Let us note that in our present analysis the quadratic constraints have been simplified as compared to supergravity, since we effectively deal only with gauge groups inside

$SO(N) \ltimes \mathbb{R}^{8N}$. In contrast, the linear constraints are as in supergravity, but supplemented with further constraints. However, this does not exclude the possibility that there exist globally-supersymmetric $\mathcal{N} = 8$ theories that satisfy weaker constraints, but which cannot be obtained as limits of supergravity in the given way.

2.3 The globally supersymmetric $\mathcal{N} = 8$ theory

In this subsection we summarize the resulting globally supersymmetric action, after taking the limit of $\mathcal{N} = 8$ gauged supergravity[5] as defined in the previous section.

The Lagrangian of the globally supersymmetric theory is given by

$$\begin{aligned}\mathcal{L} = &+\frac{1}{2} D_\mu X^{Ia} D^\mu X^{Ia} - ie\,\bar{\chi}^{\dot{A}a}\gamma^\mu D_\mu \chi^{\dot{A}a} - \frac{1}{2}\Theta_{\alpha\beta}\varepsilon^{\mu\nu\rho} A_\mu{}^\alpha \left(\partial_\nu A_\rho{}^\beta + \frac{1}{3}\Theta_{\gamma\delta}\bar{f}^{\beta\delta}{}_\epsilon A_\nu{}^\gamma A_\rho{}^\epsilon\right)\\ &+ A_3^{\dot{A}a,\dot{B}b}\bar{\chi}^{\dot{A}a}\chi^{\dot{B}b} - V\,.\end{aligned} \tag{2.33}$$

Here the covariant derivatives of the scalars and fermions are

$$\begin{aligned} D_\mu X^{aI} &= \partial_\mu X^{aI} + \Theta_{ab,cd} A_\mu{}^{cd} X^{bI} + \frac{1}{2}\Theta_{aI,bc} A_\mu{}^{bc} + \Theta_{ab,cJ} A_\mu{}^{cJ} X^{bI} + \Theta_{aI,bJ} A_\mu{}^{bJ}\,,\\ D_\mu \chi^{\dot{A}a} &= \partial_\mu \chi^{\dot{A}a} + \mathcal{Q}_\mu{}^{ab}\chi^{\dot{A}b}\,, \qquad \mathcal{Q}_\mu{}^{ab} = \frac{1}{2}\Theta_{ab,cd} A_\mu{}^{cd} + \Theta_{ab,cI} A_\mu{}^{cI} \end{aligned} \tag{2.34}$$

while the different Yukawa couplings are given by[6]

$$\begin{aligned} A_2^{A,\dot{A}a} &= -\frac{1}{12}\Gamma^{IJK}_{A\dot{A}}(-\Theta_{ab,cd} X^b{}_K X^c{}_I X^d{}_J - \Theta_{aK,bc} X^b{}_I X^c{}_J + 2\Theta_{ab,c[I} X^c{}_{J]} X^b{}_K +\\ &\qquad + 2\Theta_{aK,b[I} X^b{}_{J]} + \Theta_{IJ,KL} X^L{}_a)\,,\\ A_3^{\dot{A}a,\dot{B}b} &= +\frac{1}{48}\delta^{ab}\Gamma^{IJKL}_{\dot{A}\dot{B}}\Theta_{IJ,KL} + \frac{1}{2}\Gamma^{IJ}_{\dot{A}\dot{B}}(-\Theta_{ab,cd} X^c{}_I X^d{}_J + 2\Theta_{ab,c[I} X^c{}_{J]})\,, \end{aligned} \tag{2.35}$$

The scalar potential is positive definite in the global case and reads

$$V = \frac{1}{4} A_2^{A,\dot{A}a} A_2^{A,\dot{A}a}\,. \tag{2.36}$$

The Lagrangian is invariant under the following global $\mathcal{N} = 8$ supersymmetry transformations:

$$\begin{aligned} \delta_\epsilon X^{Ia} &= \bar{\epsilon}^A \Gamma^I_{A\dot{A}}\chi^{\dot{A}a}\,, & \delta_\epsilon \chi^{\dot{A}a} &= \frac{1}{2} i\Gamma^I_{A\dot{A}}\gamma^\mu \epsilon^A D_\mu X^{Ia} + A_2^{A,\dot{A}a}\epsilon^A\,,\\ \delta_\epsilon A_\mu^{ab} &= -2i\,\bar{\epsilon}^A \Gamma^I_{A\dot{A}}\gamma_\mu X^{I[a}\chi^{b]\dot{A}}\,, & \delta_\epsilon A_\mu^{aI} &= i\,\bar{\epsilon}^A \Gamma^I_{A\dot{A}}\gamma_\mu \chi^{\dot{A}a}\,, \end{aligned} \tag{2.37}$$

provided that the linear constraints implied by (2.24) and the quadratic constraints (2.26), are satisfied.

To summarize, for any choice of the embedding tensor components given in (2.24) that satisfy the quadratic constraints (2.26) we obtain a consistent globally supersymmetric $\mathcal{N} = 8$ theory. The physical interpretation of the different representations are quite different.

[5] We will omit overall κ-dependences w.r.t. the supergravity expressions, as these will drop out.

[6] See, however, the provision made in footnote 2.

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component	gauge vector	gauging	mass dim.	V	interpretation
f_{abcd}	A_μ^{ab}	SO(N)	$(0,1)$	X^6	CS gauging
f_{abcI}	A_μ^{aI}	SO(N)	$(\frac{1}{2},\frac{1}{2})$	X^4	YM gauging
	A_μ^{ab}	$\mathbb{R}^{8N}$	$(\frac{1}{2},1)$		
f^+_{IJKL}	$-$	$-$	$(1,-)$	X^2	massive CS
h_{IJ}	A_μ^{aI}	$\mathbb{R}^{8N}$	$(1,\frac{1}{2})$	X^2	top. mass. YM

Table 1: The different $SO(8) \times SO(N)$ representations of the embedding tensor that survive the limit of global supersymmetry, and the resulting gauging and gauge vectors (if applicable). The next columns indicate the mass dimensions of the $\Theta_{\alpha\beta}$ and $A_\mu{}^\gamma$ components and the order of the resulting scalar potential. The interpretation of the different models will be put forward in the next sections.

We will illustrate this with a few examples in the next section. For the moment we note that an understanding of what these different representations signify can be obtained from the covariant derivatives of the scalars and fermions (2.34). From these one can infer that f_{abcd} induces a compact SO(N) gauging, while h_{IJ} gauges the non-compact translations $\mathbb{R}^{8N}$. The representation f_{abcI} corresponds to a semi-direct product of compact and non-compact gaugings in $SO(N) \ltimes \mathbb{R}^{8N}$. Finally, the representation f_{IJKL} drops out from the covariant derivatives and therefore is a massive deformation instead of a gauging. For more information, see table 1.

Note that for our choice of scalings the R-symmetry SO(8) is never gauged; the components that give rise to the R-symmetry gaugings in supergravity either drop out or are massive deformations. Furthermore, from the table we conclude that only f_{abcd} can give rise to a conformally invariant theory with a sextet potential. In the next sections we will consider the various representations separately.

3. World-volume actions for multiple membranes

In this section we consider different examples of globally supersymmetric $\mathcal{N}=8$ theories obtained from gauged supergravity in order to illustrate the different possible gaugings outlined in the previous section (see the table). These can be interpreted as different world-volume actions for multiple 2-branes. An overview of our conventions can be found in appendix A.

3.1 Conformal gaugings and multiple M2-branes

In view of their applications to multiple M2-brane actions we first consider the conformal gaugings with $f_{abcd} \neq 0$.

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3.1.1 Bagger-Lambert theory

To reproduce the Bagger-Lambert theory we choose [39]:

$$\Theta_{ab,cd} \; = \; 2\, f_{abcd} \;, \qquad\qquad f_{abcd} = f_{[abcd]} \,. \tag{3.1}$$

This provides a particular solution of the linear constraints, while the quadratic constraint reduces to the fundamental identity of [2]. The Lagrangian reads

$$\begin{aligned}\mathcal{L} = {} & \frac{1}{2} D_\mu X^{Ia} D^\mu X^{Ia} - i\bar{\chi}^{\dot{A}a}\gamma^\mu D_\mu \chi^{\dot{A}a} + f_{abcd}\Gamma^{IJ}_{\dot{A}\dot{B}} X^c_I X^d_J \bar{\chi}^{\dot{A}a}\chi^{\dot{B}b} \\ & -\frac{1}{4}\varepsilon^{\mu\nu\rho} f_{abcd} A_\mu{}^{ab}\left(\partial_\nu A_\rho{}^{cd} + \frac{2}{3} f^d{}_{efh} A_\nu{}^{ef} A_\rho{}^{ch}\right) - V \,,\end{aligned} \tag{3.2}$$

where the covariant derivatives are given by

$$D_\mu X^{aI} \; = \; \partial_\mu X^{aI} + A_\mu{}^{cd} f_{cdab} X^{bI} \;, \qquad D_\mu \chi^{\dot{A}a} \; = \; \partial_\mu \chi^{\dot{A}a} + A_\mu{}^{cd} f_{cdab} \chi^{\dot{A}b} \,. \tag{3.3}$$

This is equivalent to the Bagger-Lambert action [2]. The supersymmetry variations (2.10) reduce to

$$\begin{aligned}\delta_\epsilon X^{aI} &= \bar{\epsilon}^A \Gamma^I_{A\dot{A}} \chi^{\dot{A}a} \,, \\ \delta_\epsilon \chi^{\dot{A}a} &= \frac{i}{2}\Gamma^I_{A\dot{A}} \gamma^\mu \epsilon^A D_\mu X^{Ia} + \frac{1}{6} f_{abcd} \Gamma^{IJK}_{A\dot{A}} X^b{}_I X^c{}_J X^d{}_K \epsilon^A \,, \\ \delta_\epsilon A_\mu{}^{ab} &= -2i\bar{\epsilon}^A \Gamma^I_{A\dot{A}} \gamma_\mu X^{I[a}\chi^{b]\dot{A}} \,,\end{aligned} \tag{3.4}$$

in agreement with the superconformal symmetry of [2].

3.2 Non-conformal gaugings and multiple D2-branes

We now consider the non-conformal gaugings with $f_{abcI} \neq 0$. As we will see, this representation leads to supersymmetric Yang-Mills theories and multiple D2-brane actions. We first discuss the non-semi-simple gaugings triggered by this representation and next consider the non-Abelian duality that converts the resulting action into a supersymmetric Yang-Mills theory.

3.2.1 Non-semi-simple gauge groups

To construct multiple D2-brane actions with kinetic Yang-Mills terms we must consider gauge groups that are not semi-simple. Specifically, this incorporates gauge groups, whose generators are partially in the direction of the non-compact t^{aI}.

We consider the simplest case, where only $\Theta_{ab,cI}$ is non-zero. The SO(8) indices are decomposed according to $I = (i, 8)$, with $i = 1, \ldots, 7$, i.e. we are going to break the R-symmetry group to SO(7). The explicit ansatz is given by the completely antisymmetric continuation of

$$\Theta_{ab,c8} \; = \; -g_{\mathrm{YM}}\, f_{abc} \;, \tag{3.5}$$

while all other components vanish. Here, f_{abc} are the structure constants of an arbitrary N-dimensional Lie algebra with an invariant tensor (in particular, f_{abc} is totally antisymmetric) and $g_{\rm YM}$ is the Yang-Mills coupling constant which has mass dimension $\frac{1}{2}$. This ansatz gives rise to two types of gauge group generators X_α according to (2.3): either proportional to t^{ab} or t^{aI}. Denoting the former by $\mathbf{X}$ and the latter by $\mathbf{T}$, respectively, this amounts to a gauge algebra, which schematically reads

$$\big[\mathbf{X},\mathbf{X}\big] \;\subset\; \mathbf{X}\,, \qquad \big[\mathbf{X},\mathbf{T}\big] \;\subset\; \mathbf{T}\,, \qquad \big[\mathbf{T},\mathbf{T}\big] \;=\; 0\,. \tag{3.6}$$

More precisely, this describes a semi-direct product between, say, a semi-simple Lie algebra $\mathfrak{g}$ with structure constants f_{abc} and the $\dim(\mathfrak{g})$ abelian translations $\mathbf{T}$.

In order to verify that (3.5) gives rise to a consistent gauging, we have to check the quadratic constraints. Following the discussion in the previous section it turns out that the only surviving quadratic constraint components are $\mathcal{Q}_{aI,bJ,cd}$ leading to the constraints

$$\Theta_{aI,c}{}^{g}\Theta_{gd,bJ} - \Theta_{aI,d}{}^{g}\Theta_{gc,bJ} - \Theta_{aI,gb}\Theta^{g}{}_{J,cd} \;=\; 0\,. \tag{3.7}$$

For the ansatz (3.5), this is satisfied by virtue of the Jacobi identities for f_{abc}, where we assume that its invariant tensor is given by δ_{ab}, possibly after a suitable change of basis. We conclude that we can gauge an arbitrary Lie group.

3.2.2 Multiple D2-branes through non-abelian duality

The world-volume theories of multiple D2-branes are known to be Yang-Mills gauge theories — as opposed to the Chern-Simons gauge theories discussed above — and are not conformally invariant. In fact, these two features are related, since in a non-abelian Yang-Mills term the gauge coupling constant needs to be dimensionful in $D \neq 4$, thus breaking the conformal invariance. In contrast, in the Chern-Simons gauge theories the gauge coupling can be chosen to be dimensionless.

To make contact with multiple D2-brane actions we now apply the non-Abelian duality of [28] converting the Yang-Mills Chern-Simons term into a standard Yang-Mills kinetic term. We use the ansatz (3.5) for the embedding tensor, where we may think of the structure constants as defining SU(N). There are two types of scalars, X^{ai} ($i = 1, \ldots, 7$) and $\bar{X}^a = X^{a8}$, for which the covariant derivatives read

$$\begin{aligned} D_\mu X^{ai} \;&=\; \partial_\mu X^{ai} + g_{\rm YM} f^{a}{}_{bc} A_\mu{}^{b} X^{ci}\,, \\ D_\mu \bar{X}^{a} \;&=\; \partial_\mu \bar{X}^{a} + g_{\rm YM} f^{a}{}_{bc} A_\mu{}^{b} \bar{X}^{c} + B_\mu{}^{a}\,, \end{aligned} \tag{3.8}$$

where we defined

$$A_\mu{}^{a} \;\equiv\; A_\mu{}^{a8}\,, \qquad B_\mu{}^{a} \;\equiv\; \frac{1}{2}\Theta_{a8,bc} A_\mu{}^{bc}\,. \tag{3.9}$$

The resulting action reads

$$\begin{aligned} \mathcal{L}_{\rm D2} = &\frac{1}{2} D_\mu X^{ia} D^\mu X^{ia} + \frac{1}{2} D_\mu \bar{X}^{a} D^\mu \bar{X}^{a} - i\bar{\chi}^{\dot{A}a}\gamma^\mu D_\mu \chi^{\dot{A}a} \\ &-\frac{1}{2}\varepsilon^{\mu\nu\rho} B_{\mu a} F_{\nu\rho}{}^{a} + \bar{\chi}^{a} f_{abc}\Gamma^{8i} X^{ib}\chi^{c} - V\,, \end{aligned} \tag{3.10}$$

with the non-abelian field strength

$$F_{\mu\nu}{}^a = \partial_\mu A_\nu{}^a - \partial_\nu A_\mu{}^a + g_{\rm YM}\, f^a{}_{bc} A_\mu{}^b A_\nu{}^c \,. \tag{3.11}$$

The scalar potential is the quadratic expression in A_2, given by

$$A_2^{A,\dot{A}a} \;=\; -\frac{1}{4}\Gamma^{ij8}_{A\dot{A}} g_{\rm YM}\, f_{abc} X^b{}_i X^c{}_j \,. \tag{3.12}$$

To see the equivalence to Yang-Mills gauge theories, we observe that $B_\mu{}^a$ enters only algebraically and it can therefore be integrated out. The Stückelberg shift symmetry on the extra scalars $\bar{X}^a$ apparent in (3.8) can be used to gauge this scalar to zero. In turn, the field equations for $B_\mu{}^a$ read $B_\mu{}^a = \frac{1}{2}\varepsilon_{\mu\nu\rho} F^{\nu\rho a}$. After reinsertion into (3.10), we obtain a supersymmetric action with Yang-Mills type kinetic term,

$$\mathcal{L}_{\rm D2} = \frac{1}{2} D_\mu X^{ia} D^\mu X^{ia} - \frac{1}{4} F^{\mu\nu a} F_{\mu\nu}{}^a - i\bar{\chi}^{\dot{A}a}\gamma^\mu D_\mu \chi^{\dot{A}a} + \bar{\chi}^a f_{abc}\Gamma^{8i} X^{ib}\chi^c - V, \tag{3.13}$$

which is the standard super-Yang-Mills action for D2-branes. This dualization converted the topological gauge vectors into propagating fields, then carrying the degrees of freedom of the scalars $\bar{X}^a$.

It is instructive to compare our results on multiple D2-brane actions with the recent proposal of [16 – 18]. These theories contain two extra scalars with wrong-sign kinetic terms and thus may lead to ghosts. In a recent development it has been pointed out that these ghosts can be avoided if a different model is used where a translational symmetry, which is present in the original theory, is gauged [19 – 21]. After gauge fixing the translational symmetry and integrating out some of the fields one ends up with a supersymmetric Yang-Mills theory. In this context, we note that starting from three-dimensional Yang-Mills theory the coupling constant $g_{\rm YM}$ can be promoted to a scalar field X_+ by replacing in the Lagrangian $\mathcal{L}_{\rm YM}$ the coupling constant $g_{\rm YM}$ by X_+ and adding to the Lagrangian a term with a Lagrange multiplier 2-form gauge field $C_{\mu\nu}$ such that we obtain the total Lagrangian[7]

$$\mathcal{L}_{\rm total} = \mathcal{L}_{\rm YM} \;+\; \varepsilon^{\mu\nu\rho}\,\partial_\mu X_+\, C_{\nu\rho}\,. \tag{3.14}$$

In a second step we define a vector field $\tilde{C}^\mu \equiv \varepsilon^{\mu\nu\rho}\, C_{\nu\rho}$ and introduce a second scalar field X_- via the Stueckelberg redefinition $\tilde{C}^\mu = C^\mu - \partial^\mu X_-$ with the corresponding shift symmetry $\delta C^\mu = \partial^\mu\lambda\,, \delta X_- = \lambda$. This leads to a gauge-equivalent Lagrangian of the form

$$\mathcal{L}_{\rm total} = \mathcal{L}_{\rm YM} \;-\; \partial_\mu X_+\big(\partial^\mu X_- - C^\mu\big) \tag{3.15}$$

which is of the type considered in [19 – 21].

3.3 Massive deformations

It is well-known that background fluxes may lead to massive deformations of the worldvolume theory. Two exampes will be discussed here: the first is triggered by a four-form flux in M-theory and was recently considered in [13, 14], while the second is known to arise if the mass parameter of IIA supergravity is turned on [42]. We will show how these massive deformations also fall in the framework of section 2.3.

[7] More generally, we may replace the full embedding tensor by a set of scalar fields, see [39].

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3.3.1 Massive Bagger-Lambert theory

To reproduce the massive deformation of [13, 14] we choose

$$\Theta_{IJ,KL} = \mu\left(\epsilon_{\bar{I}\bar{J}\bar{K}\bar{L}}\,, \epsilon_{\underline{I}\underline{J}\underline{K}\underline{L}}\right)\,, \qquad \Theta_{ab,cd} = 2\,f_{abcd}\,, \qquad f_{abcd} = f_{[abcd]}\,, \tag{3.16}$$

where μ is a mass parameter and we have written the SO(8) index I as $I = (\bar{I}, \underline{I})$ in terms of SO(4) × SO(4) indices. The components $\Theta_{IJ,KL}$ are self-dual, i.e. $\Theta^{-}_{IJ,KL} = 0$, which is consistent with the linear constraints. Note that the SO(8) and SO(N) sectors decouple in the quadratic constraints.

With respect to the Bagger-Lambert theory the Yukawa coupling A_2 in (2.35) contains an extra term proportional to μ. Since the potential is quadratic in A_2 we have two further terms in the potential: one mass term quadratic in μ and X and one flux term quartic in X and linear in μ. This precisely reproduces the results of [13, 14].

3.3.2 Topologically massive D2-branes

We next consider an embedding tensor given by the symmetric-traceless tensor in (2.13) with respect to SO(8). There are several possible solutions as, for instance,

$$h_{88} \;=\; 1\,, \qquad h_{ij} \;=\; -\frac{1}{7}\delta_{ij}\,, \tag{3.17}$$

where we have split again the indices according to $I = (i, 8)$. This breaks the R-symmetry to SO(7). Therefore, it might be interpreted as a D2 brane action in a massive IIA background, which is known to lead to topologically massive vectors on the world-volume [42]. Instead of constructing a specific model, we are going to show that such a gauging generically leads to topologically massive gauge vectors. The excited components are $\Theta_{aI,bJ}$ and $\Theta_{IJ,KL}$. The latter component does not appear in covariant derivatives, neither in Chern-Simons terms, after consistently setting ${A_\mu}^{IJ} = 0$. For the bosonic couplings we therefore focus on the covariant derivatives

$$D_\mu \bar{X}^{aI} \;=\; \partial_\mu \bar{X}^{aI} + m{A_\mu}^{aI}\,, \tag{3.18}$$

where ${A_\mu}^{aI} = \Theta_{aI,bJ}{A_\mu}^{bJ}$ and we have pulled out a mass parameter m, in accordance with the mass dimension of Θ. In the limit, the Chern-Simons term reduces to an abelian term, such that one finds in total

$$\mathcal{L}_{\text{mD2}} \;=\; \frac{1}{2}D_\mu X^{aI} D^\mu X^{aI} - \frac{1}{2}m\varepsilon^{\mu\nu\rho}{A_\mu}^{aI}\partial_\nu {A_\rho}^{aI} + \cdots\,, \tag{3.19}$$

where we focused on the relevant couplings. It would be interesting the inspect the supersymmetry in more detail. However, due to the provision expressed in footnote 2, we postpone this to later work. Using the shift symmetry, we can gauge-fix X^{aI} to zero, after which the equations of motion for the gauge vectors read

$$\varepsilon^{\mu\nu\rho}\partial_\nu {A_\rho}^{aI} \;=\; mA^{\mu aI}\,. \tag{3.20}$$

This describes one massive spin-1 degree of freedom [43], which follows from the fact that (3.20) implies the two equations

$$\partial^\mu F_{\mu\nu}{}^{aI} \; = \; -\frac{1}{2} m \varepsilon_{\nu\rho\lambda} F^{\rho\lambda aI} \; = \; -m^2 A_\nu{}^{aI} \, , \qquad \partial^\mu A_\mu{}^{aI} \; = \; 0 \, . \tag{3.21}$$

An equivalent description of a single massive degree of freedom carried by a vector is given by the sum of a Maxwell term and a Chern-Simons term. Both provide a gauge-invariant description of massive vectors, which is a peculiarity in three dimensions. Furthermore, it can be checked that the quadratic constraints allow for the inclusion of $\Theta_{ab,cI}$ in addition to the symmetric traceless component. With the Ansatz (3.5) the interpretation of this combination is clear: this is equivalent to a topologically massive Yang-Mills theory.

4. Conclusions

In this work we derived a general framework for constructing gaugings and massive deformations of $\mathcal{N} = 8$ (conformal and non-conformal) supersymmetric gauge theories that describe multiple membranes. Our starting point was gauged $\mathcal{N} = 8$ supergravity in three dimensions. Performing the limit of global supersymmetry and making different choices for the embedding tensor we were able to reproduce a variety of membrane actions.

In particular, we have shown that the conformal gaugings, triggered by the anti-symmetric SO(N) representation f_{abcd}, led to the Bagger-Lambert theory describing conformal invariant multiple M2-brane actions. The non-conformal gaugings, triggered by the f_{abcI} representation, led to multiple D2-brane actions. Finally, the self-dual anti-symmetric SO(8) representation f^+_{IJKL} (in combination with f_{abcd}) led to a massive deformation of the Bagger-Lambert theory, whereas the symmetric traceless h_{IJ} representation led to a topologically massive gauge theory.

We would like to stress that in addition to these known results, our results also allow one to combine the different ingredients (subject to the quadratic constraints). This would lead to generalisations of the previously discussed theories, whose membrane interpretation might be worth investigating. It is also worthwile to investigate whether the procedure we introduced to define the limit of global supersymmetry is unique or whether other limits are possible.

On a different note, in this paper we showed how starting from a gauged supergravity theory a variety of globally supersymmetric theories could be constructed. It would be interesting to apply this technique to other situations as well. For instance, one could consider cases with less supersymmetry and compare with the results of [44, 45] for $\mathcal{N} = 4$ and [22, 23] for $\mathcal{N} = 6$. A distinguishing difference between the $\mathcal{N} = 8$ and $\mathcal{N} = 6$ cases is that, whereas the conformal $\mathcal{N} = 8$ embedding tensor can only be defined as copies of the 4-index Levi-Civita symbol, i.e., with SO(4) gauge groups inside SO(N) for $N = 4k$ with k integer, the conformal $\mathcal{N} = 6$ embedding tensor can be defined for any U(N) gauge groups [29]. This is related to the fact that for $\mathcal{N} = 6$ the global symmetry group is U(N) and hence, using complex notation, the relevant embedding tensor can be expressed in terms of Kronecker delta's, for any N, instead of a Levi-Civita tensor, for special values of N. This fact has been used in the recent constructions of [22, 23].

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Finally it would be interesting to apply the procedure to construct globally supersymmetric theories out of locally supersymmetric theories in different dimensions and, in particular, to see whether some of them can be interpreted as the worldvolume theories of multiple branes.

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A. Useful relations

A.1 Conventions

We use the following notation for the different indices:

- $I, J, \ldots = 1, \ldots, 8$ for the SO(8) R-symmetry vector indices, which will be split up according to:
  - $I = (i, 8)$ with $i, j, \ldots = 1, \ldots, 7$ when the R-symmetry is broken to SO(7),
  - $I = (\bar{I}, \underline{I})$ with $\bar{I}, \bar{J} \ldots = 1, \ldots, 4$ and $\underline{I}, \underline{J} \ldots = 5, \ldots, 8$ when the R-symmetry is broken to $\mathrm{SO}(4) \times \mathrm{SO}(4)$,
- $A, B, \ldots = 1, \ldots, 8$ for the SO(8) R-symmetry spinor indices,
- $\dot{A}, \dot{B}, \ldots = 1, \ldots, 8$ for the SO(8) R-symmetry conjugate spinor indices,
- $a, b, \ldots 1, \ldots, N$ for the SO(N) fundamental indices,
- $\mathcal{I}, \mathcal{J}, \ldots = (I, a)$ for the $\mathrm{SO}(8, N)$ fundamental indices,
- $\alpha = [\mathcal{I}\mathcal{J}] = ([IJ], [ab], Ia)$ for the $\mathrm{SO}(8, N)$ adjoint indices,
- $\underline{\alpha}, \underline{\beta}, \ldots$ are flat indices corresponding to the local $\mathrm{SO}(8) \times \mathrm{SO}(N)$ action.

Note that our conventions differ from those of [39] by an SO(8) triality rotation in order to be compatible with the supergravity results of [27]. Moreover, we employ the convention that summation over the antisymmetric index pairs $[ab]$ and $[IJ]$ is accompanied by a factor of $\frac{1}{2}$.

A.2 $\mathrm{SO}(8,N)$ structures

In order to compute the various components of the Maurer-Cartan forms and of $\mathcal{V}^{\alpha}{}_{\underline{\alpha}}$ to lowest order as used in the main text, we need the explicit form of the Lie algebra $\mathfrak{so}(8,N)$. In covariant form it reads

$$\left[t^{\mathcal{I}\mathcal{J}}, t^{\mathcal{K}\mathcal{L}}\right] \;=\; 2\left(\eta^{\mathcal{I}[\mathcal{K}}t^{\mathcal{L}]\mathcal{J}} - \eta^{\mathcal{J}[\mathcal{K}}t^{\mathcal{L}]\mathcal{I}}\right)\,, \tag{A.1}$$

with the indefinite $\eta^{\mathcal{I}\mathcal{J}} = (\delta^{IJ}, -\delta^{ab})$. Splitting this in a $\mathrm{SO}(8)\times\mathrm{SO}(N)$ covariant form (and redefining $t^{ab}\to -t^{ab}$) one finds

$$\begin{aligned}
\left[t^{IJ}, t^{KL}\right] &= 2\left(\delta^{I[K}t^{L]J} - \delta^{J[K}t^{L]I}\right)\,, &\qquad \left[t^{ab}, t^{cd}\right] &= 2\left(\delta^{a[c}t^{d]b} - \delta^{b[c}t^{d]a}\right)\,,\\
\left[t^{IJ}, t^{Ka}\right] &= -2\delta^{K[I}t^{J]a}\,, & \left[t^{ab}, t^{Ic}\right] &= -2\delta^{c[a}t^{Ib]}\,,\\
\left[t^{Ia}, t^{Jb}\right] &= \delta^{IJ}t^{ab} + \delta^{ab}t^{IJ}\,.
\end{aligned} \tag{A.2}$$

This corresponds to the following structure constants

$$\begin{aligned}
f^{ab,cd}{}_{ef} &= 8\delta^{[a}{}_{[e}\delta^{b][c}\delta^{d]}{}_{f]}\,, &\quad f^{IJ,KL}{}_{PQ} &= 8\delta^{[I}{}_{[P}\delta^{J][K}\delta^{L]}{}_{Q]}\,,\\
f^{IJ,Ka}{}_{Lb} &= -2\delta^{K[I}\delta^{J]}{}_{L}\delta^{a}{}_{b}\,, & f^{ab,cI}{}_{dJ} &= -2\delta^{c[a}\delta^{b]}{}_{d}\delta^{I}{}_{J}\,,\\
f^{Ia,Jb}{}_{cd} &= \delta^{IJ}\delta^{a}{}_{[c}\delta^{b}{}_{d]}\,,
\end{aligned} \tag{A.3}$$

where we use the conventions that summation over antisymmetric indices is accompanied by a factor of $\frac{1}{2}$, i.e., $[t^{ab}, t^{cd}] = \frac{1}{2}f^{ab,cd}{}_{ef}t^{ef}$, etc.

For the computations of $\mathcal{V}^{\alpha}{}_{\underline{\alpha}}$ one has to insert (2.1) into (2.7) and use the first of the BCH relations

$$\begin{aligned}
e^{-A}Be^{A} &= B + [B,A] + \frac{1}{2!}[[B,A],A] + \frac{1}{3!}[[[B,A],A],A] + \cdots\,,\\
e^{-A}de^{A} &= dA + \frac{1}{2!}[dA,A] + \frac{1}{3!}[[dA,A],A] + \cdots\,.
\end{aligned} \tag{A.4}$$

This yields the following components

$$\begin{aligned}
\mathcal{V}^{ab}{}_{cd} &= 2\delta^{a}{}_{[c}\delta^{b}{}_{d]} + 2\delta^{[a}{}_{[c}X^{b]I}X^{I}_{d]} + \mathcal{O}(X^4)\,,\\
\mathcal{V}^{ab}{}_{cI} &= 2\delta^{[a}{}_{c}X^{b]}{}_{I} - X^{[a}{}_{I}X^{b]J}X^{J}{}_{c} - \frac{1}{3}X^{[a}{}_{J}\delta^{b]}{}_{c}X^{Jd}X^{d}{}_{I} + \mathcal{O}(X^4)\,,\\
\mathcal{V}^{ab}{}_{IJ} &= -2X^{[a}{}_{I}X^{b]}{}_{J} + \mathcal{O}(X^4)\,,\\
\mathcal{V}^{IJ}{}_{ab} &= -2X^{I}{}_{[a}X^{J}{}_{b]} + \mathcal{O}(X^4)\,,\\
\mathcal{V}^{IJ}{}_{KL} &= 2\delta^{I}{}_{[K}\delta^{J}{}_{L]} + 2\delta^{[I}{}_{[K}X^{J]a}X^{a}{}_{L]} + \mathcal{O}(X^4)\,,\\
\mathcal{V}^{IJ}{}_{Ka} &= -2X^{[I}{}_{a}\delta^{J]}{}_{K} - X^{[I}{}_{a}X^{J]b}X^{b}{}_{K} - \frac{1}{3}X^{L}{}_{a}X^{Lb}X^{b[I}\delta^{J]}{}_{K} + \mathcal{O}(X^4)\,,\\
\mathcal{V}^{Ia}{}_{bc} &= -2X^{I}{}_{[b}\delta^{a}{}_{c]} - \frac{2}{3}X^{I}{}_{[b}X^{J}{}_{c]}X^{Ja} + \mathcal{O}(X^4)\,,\\
\mathcal{V}^{Ia}{}_{Jb} &= \delta^{I}{}_{J}\delta^{a}{}_{b} + 2X^{a[I}X^{J]}{}_{b} + \mathcal{O}(X^4)\,,\\
\mathcal{V}^{Ia}{}_{JK} &= -2X^{a}{}_{[J}\delta^{I}{}_{K]} - X^{Ib}X^{a}{}_{[J}X^{b}{}_{K]} + \mathcal{O}(X^4)\,.
\end{aligned} \tag{A.5}$$

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For the various components of the Maurer-Cartan forms (2.2) one finds by use of the second of the BCH formulas (A.4)

$$\begin{aligned}
\mathcal{Q}_\mu{}^{ab} &= \partial_\mu X^{I[a} X^{b]I} + \Theta_{\alpha\beta} A_\mu{}^\alpha \mathcal{V}^\beta{}_{ab} + \mathcal{O}(X^3) , \\
\mathcal{Q}_\mu{}^{IJ} &= \partial_\mu X^{a[I} X^{J]a} + \Theta_{\alpha\beta} A_\mu{}^\alpha \mathcal{V}^\beta{}_{IJ} + \mathcal{O}(X^3) , \\
\mathcal{P}_\mu{}^{Ia} &= D_\mu X^{Ia} + \mathcal{O}(X^3) ,
\end{aligned} \tag{A.6}$$

where the covariant derivative reads

$$D_\mu X^{aI} = \partial_\mu X^{aI} + \Theta_{\alpha\beta} A_\mu{}^\alpha \mathcal{V}^\beta{}_{Ia} . \tag{A.7}$$

Furthermore we expand the T-tensor:

$$\begin{aligned}
T_{IJ,KL} &= -\Theta_{ab,IJ} X^a{}_K X^b{}_L + 4\Theta_{aI,bK} X^a{}_J X^b{}_L + 2\Theta_{aI,KL} X^a{}_J + ([IJ] \leftrightarrow [KL])\big) \\
&\quad + \Theta_{IJ,KL} + \mathcal{O}(X^3) , \\
T_{IJ,Ka} &= -\Theta_{ab,cd} X^b{}_K X^c{}_I X^d{}_J + \Theta_{bc,LK} X^b{}_I X^c{}_J X^L{}_a \\
&\quad - \frac{1}{2}\Theta_{IJ,bc} X^b{}_K X^{cL} X^L{}_a + \frac{1}{6}\Theta_{IJ,ab} X^{Lb} X^{Ld} X^d{}_K \\
&\quad - \frac{1}{2}\Theta_{IJ,PQ} X^P{}_a X^{Qb} X^b{}_K + \frac{1}{6}\Theta_{IJ,KL} X^P{}_a X^{Pb} X^{Lb} \\
&\quad - \Theta_{aK,bc} X^b{}_I X^c{}_J + 2\Theta_{ab,c[I} X^c{}_{J]} X^b{}_K + 2\Theta_{KL,b[I} X^b{}_{J]} X^L{}_a \\
&\quad + 2\Theta_{aK,b[I} X^b{}_{J]} + \Theta_{IJ,ab} X^b{}_K + \Theta_{IJ,KL} X^L{}_a + \Theta_{IJ,Ka} + \mathcal{O}(X^4) , \\
T_{IJ,ab} &= -\Theta_{ab,cd} X^c{}_I X^d{}_J - \Theta_{IJ,KL} X^K{}_a X^L{}_b \\
&\quad + 2\Theta_{ab,c[I} X^c{}_{J]} + 2\Theta_{IJ,K[a} X^K{}_{b]} - 4X^c{}_{[I}\Theta_{J]c,K[a} X^K{}_{b]} + \Theta_{IJ,ab} + \mathcal{O}(X^3) ,
\end{aligned} \tag{A.8}$$

where we suppressed in the first line an antisymmetrization in $[IJ]$ and $[KL]$.

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